Robust three-axis attitude stabilization for inertial pointing spacecraft using magnetorquers

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Outline

• three-axis inertial pointing
• model for spacecraft and geomagnetic field
• spacecraft controllability
• attitude and attitude rate feedback
• attitude only feedback
Three-axis inertial pointing

spacecraft on a circular Low Earth Orbit

Earth Centered Inertial frame

body frame

attitude of body frame with respect to inertial frame parametrized by quaternion

\[ q = [q_1 \ q_2 \ q_3 \ q_4]^T = [q_v \ q_4] \quad q_v \text{ vector part of } q \]

attitude matrix \( A(q) \)

three-axis inertial pointing

\[ A(q) = I_3 \Leftrightarrow q_v = 0 \quad q_4 = \pm 1 \]
Spacecraft model

relative kinematics

\[
\begin{align*}
\dot{q}_v &= \frac{1}{2} (q_v \times \omega + q_4 \omega) \\
\dot{q}_4 &= -\frac{1}{2} q_v \omega
\end{align*}
\]

\(\iff \dot{q} = W(q)\omega\)

dynamics

\[
J \dot{\omega} = -\omega \times J \omega + T
\]

\(J\) is **uncertain** but bounds \(J_{\text{min}}\) and \(J_{\text{max}}\) on principal moments of inertia are known

spacecraft is equipped with three coils aligned with body axis

\(m_{\text{coils}}\) vector of coils’ magnetic moments

\[
T_{\text{coils}} = m_{\text{coils}} \times B^b \implies T_{\text{coils}} \perp B^b
\]

\[
B^b(q, t) = A(q)B^i(t)
\]

\(B^i\) geomagnetic field in inertial frame

spacecraft model

\[
\begin{align*}
\dot{q} &= W(q)\omega \\
J \dot{\omega} &= -\omega \times J \omega - (A(q)B^i(t)) \times m_{\text{coils}}
\end{align*}
\]
Dipole model of geomagnetic field

\[ B^i(t) \] geomagnetic field at spacecraft in inertial frame

\[ B^i(t) \] is the sum of sinusoidal functions having different frequencies (vector of almost periodic functions)

**spacecraft model**

\[
\begin{align*}
\dot{q} &= W(q)\omega \\
J\dot{\omega} &= -\omega \times J\omega - (A(q)B^i(t)) \times m_{coils}
\end{align*}
\]

nonlinear almost periodic system
Control design

spacecraft model

\[ \begin{align*}
\dot{q} &= W(q)\omega \\
J\dot{\omega} &= -\omega \times J\omega - (A(q)B^i(t)) \times m_{\text{coils}}
\end{align*} \]

nonlinear almost periodic system

objective: design control law for \( m_{\text{coils}} \) so that \( q_v \to 0 \) and \( \omega \to 0 \)

preliminary control

\[ m_{\text{coils}} = B^b \times u \]

\( B^b \) can be measured (magnetometers) \quad \( u \in \mathbb{R}^3 \) new control vector

preliminary control \quad \Rightarrow \quad m_{\text{coils}} \perp B^b 

\[ \downarrow \]

spacecraft model

\[ \begin{align*}
\dot{q} &= W(q)\omega \\
J\dot{\omega} &= -\omega \times J\omega + A(q)\Gamma^i(t)A(q)^T u
\end{align*} \]

\( \Gamma^i(t) = B^i(t)^T B^i(t)I - B^i(t)B^i(t)^T \quad \Gamma^i(t) \in \mathbb{R}^{3 \times 3} \) almost periodic

\[ \det(\Gamma^i(t)) = 0 \quad \forall t \quad \Rightarrow \quad \text{spacecraft is not controllable at each } t \]
Average controllability

spacecraft model

\[
\begin{align*}
\dot{q} &= W(q)\omega \\
J\dot{\omega} &= -\omega \times J\omega + A(q)\Gamma^i(t)A(q)^T u
\end{align*}
\]

\[
\Gamma^i(t) = B^i(t)^T B^i(t)I - B^i(t)B^i(t)^T \quad \Gamma^i(t) \in \mathbb{R}^{3 \times 3} \quad \text{almost periodic}
\]

\[
det(\Gamma^i(t)) = 0 \quad \forall t \quad \Rightarrow \quad \text{system is not controllable}
\]

average of \( \Gamma^i(t) \)

\[
\Gamma^{i}_{av} = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} \Gamma^i(\tau)d\tau
\]

**average controllability assumption** - spacecraft's orbit satisfies \( det(\Gamma^{i}_{av}) \neq 0 \)

average controllability assumption is satisfied for all circular orbits except equatorial orbit
Robust attitude plus attitude rate feedback

spacecraft model

\[
\begin{align*}
\dot{q} &= W(q)\omega \\
J\dot{\omega} &= -\omega \times J\omega + A(q)\Gamma^i(t)A(q)^T u
\end{align*}
\]

attitude plus attitude rate feedback

\[u = -(\epsilon^2 k_1 q_v + \epsilon k_2 \omega)\]  \hspace{1cm} \text{(PD-like feedback)}

\[k_1 > 0 \quad k_2 > 0 \quad 0 < \epsilon < \epsilon^* \]

\[\downarrow\]

\[q_v = 0 \quad q_4 = 1 \quad \omega = 0\]  \hspace{1cm} \text{locally exponentially stable}

for all \(J\)'s with principal moments of inertia in the range \([J_{\text{min}} J_{\text{max}}]\)
Case study

\[
J = \begin{bmatrix}
5 & -0.1 & -0.5 \\
-0.1 & 2 & 1 \\
-0.5 & 1 & 3.5
\end{bmatrix} \text{ kg m}^2
\]

circular orbit \hspace{1cm} \text{inclination } 87^\circ \hspace{1cm} \text{altitude } 450 \text{ km}

\[\downarrow\]
average controllability

rest-to-rest maneuver

• initial attitude \( \phi(0) = 0.1 \text{ rad} \quad \theta(0) = 0.2 \text{ rad} \quad \psi(0) = 0.3 \text{ rad} \quad \omega(0) = 0 \)

• desired final attitude \( \phi = \theta = \psi = \omega = 0 \)
Case study – state feedback

\[ m_{\text{coils}} = -B^b \times (\epsilon^2 k_1 q_v + \epsilon k_2 \omega) \quad k_1 = 6 \times 10^8 \quad k_2 = 2 \times 10^9 \quad \epsilon = 10^{-3} \]
Case study - robustness

\[ J = \begin{bmatrix} 5 & -0.1 & -0.5 \\ -0.1 & 2 & 1 \\ -0.5 & 1 & 3.5 \end{bmatrix} \text{ kg m}^2 \]

principal moments of inertia of \( J \) \[
\begin{cases} 
1.4947 \\
3.7997 \\
5.2056
\end{cases} \text{ kg m}^2
\]

\[ J_{\text{pert}} = \begin{bmatrix} 1.4947 & 0 & 0 \\ 0 & 3.7997 & 0 \\ 0 & 0 & 5.2056 \end{bmatrix} \text{ kg m}^2 \]

robust state feedback
Case study – attitude and attitude rate feedback

\[ m_{coils} = -B^b \times (\epsilon^2 k_1 q_v + \epsilon k_2 \omega) \quad k_1 = 6 \times 10^8 \quad k_2 = 2 \times 10^9 \quad \epsilon = 10^{-3} \]
Robust attitude only feedback

spacecraft model
\[
\begin{align*}
\dot{q} &= W(q)\omega \\
J\dot{\omega} &= -\omega \times J\omega + A(q)\Gamma^i(t)A(q)^T u
\end{align*}
\]

attitude only feedback - no rate gyros
\[
\begin{align*}
\dot{\delta} &= \alpha(q - \epsilon\lambda\delta) \\
u &= -\epsilon^2 \left( k_1 q_v + k_2 \alpha \lambda W(q)^T (q - \epsilon\lambda\delta) \right) \\
\delta &\in \mathbb{R}^4
\end{align*}
\]

\[k_1 > 0 \quad k_2 > 0 \quad \alpha > 0 \quad \lambda > 0 \quad 0 < \epsilon < \epsilon^*
\]
\[
\downarrow
\]
\[q_v = 0 \quad q_4 = 1 \quad \omega = 0 \quad \text{locally exponentially stable}
\]

for all $J$'s with principal moments of inertia in the range $[J_{min}, J_{max}]$
Case study – attitude only feedback

\[ k_1 = 7 \times 10^8 \quad k_2 = 10^{10} \quad \alpha = 10^3 \quad \lambda = 1 \quad \epsilon = 10^{-3} \]

\[ J \]

\[ J_{pert} \]
Conclusions and way forward

• three-axis stabilization for inertial pointing spacecraft using magnetorquers
• attitude and attitude rate feedback
• attitude only feedback
• local exponential stabilization and robustness with respect to uncertainty on inertia matrix
• Earth-pointing case
• global stabilization